FORMATION OF STRUCTURE IN A FLOW OF A SUSPENSION OF NEUTRALLY FLOATING PARTICLES

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It is shown that in a flow with a nonuniform vorticity, the concentration field of the suspension is also nonuniform. Stationary Couette and Poiseuille flows in a flat gap and in a capillary are examined in detail.

It is well known that when a suspension of fine particles flows in a channel, there is a redistribution of particles over the cross section of the channel, as a result of which an inhomogeneous concentration distribution of the suspension forms in the flow (see, for example, [1-3]). The latter has a large effect on the observed properties of the flow, for example, on the values of the effective viscosity, measured in capillaries with viscosimeters.

At the present time neither the detailed characteristics of stationary concentration distributions, established under different conditions, nor the fundamental methods for determining them experimentally, nor even the basic physical reasons responsible for the formation of structure are known. To explain this formation from a qualitative point of view, assumptions are usually made about transverse migration of individual particles in nonuniform shear flows, i.e., the so-called Segre–Silberberg effect [4], as well as about the interaction of particles with the walls bounding the flow. The indicated migration is due to the influence of inertial effects leading to the appearance of a "lifting" force acting on a single particle and proportional to the vector product of its angular rotational velocity and its translational velocity relative to the fluid (see the reviews in [3, 5] and the references therein). However, arguments of this kind are not directly applicable to flow of a suspension, since they do not take into account the collective interaction of the system of suspended particles and, in particular, the screening effect of all particles on any isolated particle. In addition, distinct structure formation also occurs when the average phase velocities of the suspension coincide, i.e., the lifting forces generally vanish [1, 3].

The known attempts to describe the real velocity and concentration profiles in one-dimensional stationary flows of suspensions have the typical phenomenological character. They are usually based on postulation of some extremal principle (for example, principle of minimum dissipation of energy in the flow [6, 7]) or they are related to a priori and, generally speaking, unjustified introduction of an asymmetric effective stress tensor, as well as reversible and irreversible processes of aggregation, diffusion, and so on using the methods of linear thermodynamics of irreversible processes [8, 9].

In what follows, the formation of structure in a suspension with a nonuniform shear flow is examined using the general continuum mechanics of concentrated dispersed systems, developed in [10-12]. For simplicity, the analysis is restricted to stable uniform-density suspensions of identical fine spherical particles in the absence of aggregation and any inertial and fluctuation effects. In addition, it is demonstrated that the basic physical mechanism for the formation of structure may not be related to such effects at all, but rather to the necessity of conserving the internal angular momentum of the continuous phase of the suspension, which must be viewed as a completely independent property of the flow [13].

Equations and Boundary Conditions. The system of equations for conservation of mass, momentum, and angular momentum of the continuous and dispersed phases of the suspension, which we assume to be incompressible and examine as coexisting interpenetrating continua, are formulated in [10-12]. These equations serve to determine the bulk concentration of particles, the pressure of the dispersed medium, the average phase velocities of the suspension and their angular momenta (for the discrete phase, the internal angular momentum is proportional to the angular velocity of the particles). Here we examine only one-dimensional stationary flows, in which all of these quantities, except for the pressure, are independent of time and the coordinate x in the direction of motion, while ∇p is constant and oriented along x. If the particles

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have zero buoyancy, then, neglecting the influence of the interphase interaction forces on the conservation of angular momentum [10-12], the influence of the external mass field can be neglected, since the external forces can be easily included in the effective pressure. If the quantity $|\nabla p|$ is not very large, we can restrict the analysis to the single-velocity model, according to which the phase velocities are identical, which is supported by observations [1, 3]. We also assume that there are no external force couples. Then the equations of conservation are considerably simplified. The equation of conservation of momentum of the suspension as a whole and the equation of conservation of angular momentum of its continuous phase assume the form

$$\nabla\cdot\sigma=0,\ \nabla\cdot\gamma=0,\ \sigma=-p\mathbf{I}+2\mu_0 M(\rho)\,\mathbf{e}\{\mathbf{c}\},$$
$$\gamma=2a^2\mu_0\rho\Gamma(\rho)\,\mathbf{e}\{\mathrm{rot}\,\mathbf{c}\},\ \ \mathbf{e}\{\mathbf{c}\}=\frac{1}{2}\left\|\frac{\partial c_i}{\partial x_j}+\frac{\partial c_j}{\partial x_i}\right\|, \tag{1}$$

where $M(\rho)$ and $\Gamma(\rho)$ are monotonically increasing functions equal to unity at $\rho = 0$ and the operator ∇ involves differentiation only with respect to the transverse coordinates. In addition to Eqs. (1), in the case examined, the equation of conservation of internal angular momentum of the dispersed phase, which must be used to determine the angular velocity of the particles, is also not an identity; we shall not examine this equation here.

For moderately concentrated suspensions ($\rho < 0.20$–0.25), the quantities M and Γ were calculated based on the method [10-12] in [14]:

$$\Gamma(\rho)=M(\rho)=(1-5\rho/2)^{-1}, \tag{2}$$

and for suspensions with high concentration, numerical calculations of M are available for different forms of the binary particle distribution function [15]. In what follows, we shall use the well-known simple approximation

$$\Gamma(\rho)=M(\rho)=(1-\rho)^{-5/2}, \tag{3}$$

close to (2) for suspensions with moderate concentration, but not having in contrast to (2) singularities in the interval $0 \leq \rho < 1$.

For flows of the type examined, the equation of conservation of internal angular momentum of the continuous phase, characterizing the intensity of local circulatory motions of the dispersed phase due to rotation of the particles, essentially becomes degenerate in the sense that this moment itself does not enter into the indicated equation. For this reason, the latter circumstance provides some additional relation, imposed on the velocity and concentration fields. We emphasize that both equations in (1) are completely equivalent and, as will be shown in what follows, without the second equation in (1) it is impossible to explain the formation of structure of a suspension just as without the analogous equation it is impossible to describe, for example, the well-known gyromagnetic effect [13].

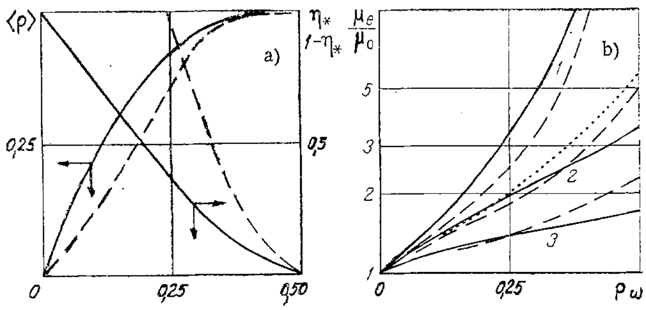

Fig. 1. Average volume concentration $\langle\rho\rangle$ and dimensionless distance from the wall to the closely packed core η_* and $1-\eta_*$ (for a two-dimensional and axisymmetrical problems, respectively) (a) and two dimensionless viscosity (b) as a function of the particle concentration at the wall ρ_w; the continuous curves are for a flow in a capillary and the dashed curves are for a flow in a flat gap; the point show the function $\mu_e/\mu_0 = (1-\rho_w)^{-5/2}$, 1-3) $m\varkappa = 0$; 0.1 and 0.2; $\rho_* = 0.50$.

In mechanics of suspensions, the equations of conservation of internal angular momenta of phases (or conservation of angular momenta), first rigorously obtained in [10], are usually not taken into account. (The only example of the effective use of these equations known to the author occurs in [16], where the formation of structure in a flow of a dilute suspension of spherical magnetic dipoles in an external magnetic field was investigated.) Instead, attempts are often made to include the influence of internal vorticity on the average (observed) characteristics of the flow by introducing into the effective stress tensor an antisymmetrical part [8, 17-19]. As shown in [11, 12], by averaging over the configuration ensemble of the system of particles in the dispersed phase, such attempts are inadequate. †

For the necessary conditions that must be imposed on the solution, we shall use the conditions following from the requirement of symmetry of the flow examined as well as the double inequality $0 \leq \rho \leq \rho_*$. On solid walls, a boundary condition of the third kind must be given for the tangential component of the velocity. This condition appears due to the impenetrability of the walls for solid particles and the resulting formation of a layer with thickness of the order of a near the wall, in which the concentration rapidly drops from ρ_W at its outer boundary to zero at the wall itself [12]. We have

$$c = k_w a \frac{dc}{dn}, \ n = 0, \ k_w = k(\rho_w), \tag{4}$$

where n is the coordinate measured from the wall along the normal to it.

If the indicated thin layer is modeled using the idea that there is a layer of thickness ma filled with a pure dispersed medium and separating the wall from the suspension itself, then it can be shown that ‡

$$k_w = m(M_w - 1), \ M_w = M(\rho_w). \tag{5}$$

The relations presented are sufficient to analyze the formation of structure in flows of the type examined with negligibly low interphase slipping velocities.

Couette Flow. In this case, Eqs. (1) after a single integration assume the form (here and in what follows we set $M = \Gamma$ in accordance with (2) or (3))

$$M(\rho)\frac{dc}{dy} = \alpha, \ \ \rho M(\rho)\frac{d^2c}{dy^2} = \beta, \tag{6}$$

where α and β are integration constants. Eliminating c, we have from (6)

$$\frac{d\rho}{dy} = -\frac{\beta}{\alpha}\frac{M}{\rho M'}, \ M' = \frac{dM}{d\rho}. \tag{7}$$

Equation (7) has a single-parameter family of solutions, satisfying the condition $\rho(0) = \rho_W$; if we use Eq. (3), then it is determined in implicit form by the relation

$$\ln\frac{1-\rho_w}{1-\rho} - (\rho - \rho_w) = -\frac{2}{5}\frac{\beta}{\alpha}y. \tag{8}$$

† The inadequate understanding of the possible ways to take into account and describe internal vorticity in flows of both single-phase and dispersed systems, unfortunately, is very widespread, which is indicated, in particular, by the discussion in [20,21], raised again by Nikolaevskii et al. [17, 18]. For this reason, it is useful to stop here to consider this in greater detail. The conclusion that there exists antisymmetrical stresses follows automatically if in obtaining the equations for the macroscopically observed motion by averaging with respect to spatial objects it is assumed that the results of averaging over areas differs from the result obtained by averaging over a small physical volume and, moreover, depends on the orientation of the areas. Actually, this conclusion is a direct result of the indicated hypothesis. Using the procedure of spatial averaging, it is impossible to determine the degree to which this hypothesis is valid in principle and for this reason discussions of whether or not antisymmetrical stresses are present in the flow is a clear example of pseudoscientific scholastics. However, a unique negative answer to this question can be obtained using the more general procedure of ensemble averages. As demonstrated in [11, 12], antisymmetrical stresses in suspensions of spherical particles appear only in the presence of external force couples acting on the particles.

‡ This method is presented in detail in the following preprint: Yu. A. Buevich, B. S. Endler, and I. N. Shchelkova, "Continuum mechanics of monodispersed suspensions. Rheological equations of state," Preprint No. 85, Institute of Applied Mechanics, Academy of Sciences of the USSR, Moscow (1977).

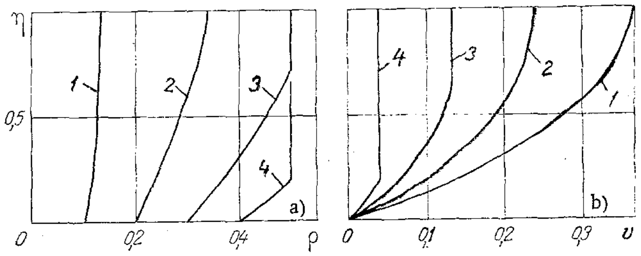

Fig. 2. Concentration (a) and velocity (b) profiles for flow in a planar gap: 1-4) $\rho_W = 0.1$; 0.2; 0.3; and 0.4; $\rho_* = 0.50$.

From the symmetry of the flow relative to the central plane y = h, it is evident that the function $\rho(y)$ must be symmetrical relative to the point y = h, i.e., it is necessary to assume that $\beta = 0$; then $\rho = \langle\rho\rangle =$ const and structure formation does not occur. Physically this is completely understandable because in the usual simple shear flow (with $\rho \equiv \langle\rho\rangle$) the vorticity field by definition is uniform, so that the tensor of angular stresses γ vanishes, while the second equation in (1) is an identity. (The concentration field described for $\beta \neq 0$ by the function $\rho(y)$ from (8) in the region $0 \le y < h$ and its even continuation into the region $h \le y \le 2h$ is not a solution of Eq. (6) with constants α and β. It is easy to see that for such a field β has a discontinuity of the first kind at y = h, which corresponds to a source of angular momentum localized on the center plane of the flow.)

Integrating the first equation in (6) with the condition (4) and including the fact that the relative velocity 2V of the bounding planes y = 0 and y = 2h is assumed to be fixed, we obtain the usual equations for the velocity profile and the tangential stress:

$$c(y) = c_0 + (V - c_0)\frac{y}{h},\quad \tau = \mu_0 M\frac{V - c_0}{h},\quad \varkappa = \frac{a}{h},$$
$$c_0 = \varkappa k(V - c_0),\quad k = k(\langle\rho\rangle),\quad M = M(\langle\rho\rangle). \tag{9}$$

As expected, the stress τ under otherwise equal conditions decreases monotonically with increasing ratio $\varkappa = a/h$.

Planar Poiseuille Flow. In examining the flow in the gaplike channel with width 2h under the action of a constant pressure gradient, it is convenient to introduce the dimensionless quantity

$$v = \frac{\mu_0 c}{h^2}\left|\frac{dp}{dx}\right|^{-1},\quad \eta = \frac{y}{h},\quad \varkappa = \frac{a}{h}, \tag{10}$$

in which (1) and (4) and other conditions are written as follows:

$$\frac{d}{d\eta}\left(M(\rho)\frac{dv}{d\eta}\right) = -1,\quad \frac{d}{d\eta}\left(\rho M(\rho)\frac{d^2v}{d\eta^2}\right) = 0,$$
$$v = k_w\varkappa\frac{dv}{d\eta}\ (\eta = 0),\quad \frac{dv}{d\eta} = 0\ (\eta = 1),\quad 0 \leqslant \rho \leqslant \rho_*. \tag{11}$$

In view of the total symmetry relative to the plane $\eta = 1$, it is sufficient to investigate the solution (11) in the region $0 \le \eta \le 1$. After a single integration we obtain from (11)

$$M(\rho)\frac{dv}{v\eta} = 1 - \eta,\quad \rho M(\rho)\frac{d^2v}{d\eta^2} = \beta. \tag{12}$$

When approaching the wall, the quantity $d^2v/d\eta^2$ must approach the value -1, characteristic of a pure dispersed medium. For this reason, from (12), we have $\beta = -\rho_W M_W$, $M_W = M(\rho_W)$. Eliminating now from (12) the quantity v and using the expression obtained for β, we arrive at an equation for ρ:

$$\frac{d\rho}{d\eta} = \left(\frac{\rho_w M_w}{\rho} - 1\right)\frac{M}{M'(1-\eta)},\quad M' = \frac{dM}{d\eta}. \tag{13}$$

Let $\rho(1) = \rho_c$. Examining (13) near the plane $\eta = 1$, we obtain the approximation for $1 - \eta \ll 1$

$$\rho_c - \rho \sim (1-\eta)^{\sigma}, \quad \frac{d\rho}{d\eta} \sim (1-\eta)^{\sigma-1}, \qquad \sigma = \frac{M_c}{\rho_c M_c'}, \quad \rho_c = \rho_w M_w. \tag{14}$$

From here it is evident that the function $\rho(\eta)$ at the point $\eta = 1$ does not have singularities, if $\sigma > 0$, and it has a zero derivative if $\sigma > 1$.

Using in what follows Eq. (3), we obtain from (13) and (14) the following implicit dependence of ρ on η:

$$\frac{\rho_c - \rho}{\rho_c - \rho_w}\left(\frac{1-\rho_w}{1-\rho}\right)^{1/\rho_c} = (1-\eta)^{\sigma}, \qquad \sigma = \frac{2}{5}\frac{1-\rho_c}{\rho_c}, \quad \rho_c = \frac{\rho_w}{(1-\rho_w)^{5/2}}, \tag{15}$$

It follows from here that $d\rho/d\eta = 0$ at $\eta = 1$ only if $\rho_c < 2/7 \approx 0.286$, i.e., $\rho_w \lesssim 0.176$, while the inequality $\sigma > 0$ is always satisfied because $\rho_c < 1$. The quantity ρ_c is less than ρ_* when ρ_w is less than some critical value ρ_w^* and becomes equal to ρ_* at $\rho_w = \rho_w^*$. If it is assumed that $\rho_* = 0.50$ (such a low value of ρ_* is apparently justified since closely packed systems, appearing in the flow, must be quite loose), then $\rho_w^* \approx 0.246$ (5).

If $\rho_w > \rho_w^*$, then for some value $\eta_* = y_*/h$, the function $\rho(\eta)$ assumes a value equal to ρ_*. In this case, a zone with thickness $2(1-\eta_*)h$, in which particles are located in the closely packed state, forms in the core of the flow.

The quantity ρ_w, playing the role of a parameter in the problem, is naturally determined from one of the conditions

$$\langle \rho \rangle = \int_0^1 \rho(\eta)\, d\eta, \quad \langle \rho \rangle = \int_0^{\eta_*} \rho(\eta)\, d\eta + (1-\eta_*)\rho_*, \tag{16}$$

valid for $\rho_w < \rho_w^*$, $\rho_w > \rho_w^*$, respectively. The $\langle\rho\rangle$ dependence of the quantities ρ_w and η_* at $\rho_* = 0.50$ are presented in Fig. 1a.

Integrating the first equation in (12) with conditions on $v(\eta)$ in (11), we obtain for $0 \le \rho_w < \rho_w^*$

$$v(\eta) = v_0 + \int_0^{\eta} \frac{1-\eta}{M(\rho)}\, d\eta, \quad q = \int_0^1 v(\eta)\, d\eta, \quad v_0 = \varkappa \frac{k_w}{M_w} \tag{17}$$

and for $\rho_w^* \le \rho_w \le \rho_*$

$$v(\eta) = \begin{cases} v_0 + \displaystyle\int_0^{\eta} \frac{1-\eta}{M(\rho)}\, d\eta, & 0 \leqslant \eta < \eta_*, \\ v_* = v_0 + \displaystyle\int_0^{\eta_*} \frac{1-\eta}{M(\rho)}\, d\eta, & \eta_* \leqslant \eta \leqslant 1, \end{cases} \tag{18}$$

$$q = \int_0^{\eta_*} v(\eta)\, d\eta + (1-\eta_*) v_*.$$

From the curves in Figs. 1a it is not difficult to obtain inequalities for $\langle\rho\rangle$, corresponding to those presented for ρ_w.

The effective viscosity μ_e, which is determined in a standard manner in experiments measuring the values of the flow rate and pressure drop based on the idea that the moving suspension is homogeneous, is of great interest. This viscosity can be expressed in dimensionless variables as follows:

$$\mu_e = \mu_0/3q. \tag{19}$$

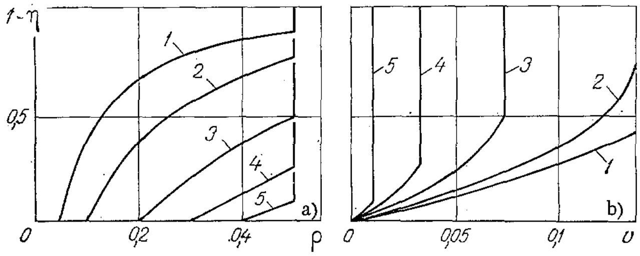

Fig. 3. Concentration (a) and velocity (b) profiles for flow in a capillary: 1-5) ρ_w = 0.05; 0.1; 0.2; 0.3 and 0.4; ρ_* = 0.50.

The dependence of the ratio μ_e/μ_0 on $<\rho>$ with $\rho_* = 0.50$ and different $m\varkappa$ is shown in Fig. 1b; Eq. (5) was used for k_w. It is evident that μ_e in the structured flow differs greatly from the magnitude $\mu = \mu_0 M(<\rho>)$ of the viscosity with particles distributed uniformly over the cross section of the channel.

The characteristic profiles $\rho(\eta)$ and $v(\eta)$ at $\varkappa = 0$, corresponding to $\rho_* = 0.50$ and different ρ_w, are presented in Fig. 2. It is evident that three types of concentration distributions are possible. For $\rho_w < 0.176$, the distribution is continuous together with its first derivative; for $0.176 < \rho_w < 0.246$ (5), this derivative is discontinuous at $\eta = 1$; for $\rho_w > 0.246$ (5), a zone with closely packed particles, which behaves as a rigid body, forms in the core of the flow.*

Axisymmetrical Poiseuille Flow. For flow in a capillary with a circular cross section, it is natural to introduce the same dimensionless quantities (10), interpreting h as the radius of the capillary. Instead of (11), we obtain in this case

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta M(\rho)\frac{dv}{d\eta}\right) = -1, \quad \frac{d}{d\eta}\left[\eta^3 \rho M(\rho)\frac{d}{d\eta}\left(\frac{1}{\eta}\frac{dv}{d\eta}\right)\right] = 0,$$
$$v = -k_w \varkappa \frac{dv}{d\eta}\ (\eta = 1), \quad \frac{dv}{d\eta} = 0\ (\eta = 0), \quad 0 \leqslant \rho \leqslant \rho_*, \tag{20}$$

and a single integration gives

$$M(\rho)\frac{dv}{d\eta} = -\frac{\eta}{2}, \quad \eta^3 \rho M(\rho)\frac{d}{d\eta}\left(\frac{1}{\eta}\frac{dv}{d\eta}\right) = \beta. \tag{21}$$

We obtain the value of the constant β, as previously, from the requirement that as the wall is approached, i.e., for $\eta \to 1$, the equation of conservation of momentum of the suspension must transform continuously into the equation for a pure dispersed medium. As a result, we obtain from (20) and (21) $\beta = -\rho_w(M_w - 1)$. Eliminating now the variable v from (21) and using the expression for β, we obtain the equation

$$\frac{d\rho}{d\eta} = -2\rho_w(M_w - 1)\frac{M}{\rho M' \eta^3}, \quad M' = \frac{dM}{d\rho}, \tag{22}$$

whose solution in implicit form with $M(\rho)$ from (3) has the form

$$\ln\frac{1-\rho_w}{1-\rho} - (\rho - \rho_w) = \frac{2}{5}\rho_w(M_w - 1)\left(\frac{1}{\eta^2} - 1\right). \tag{23}$$

The quantity $\rho(\eta)$ represents a monotonically decreasing function, with a singularity at $\eta = 0$. For this reason, in contrast to the plane Poiseuille flow, in this case, only concentration profiles of the third type are realized: for arbitrarily small $<\rho>$, a "rod" of closely packed particles with radius $\eta_* = \eta_*(<\rho>)$ forms in the core of the flow. The equation for determining the dependence of the parameter ρ_w on $<\rho>$, replacing (16), has the form

$$\langle \rho \rangle = \eta_*^2 \rho_* + 2\int_{\eta_*}^{1} \eta\rho(\eta)\,d\eta. \tag{24}$$

* We emphasize that the symmetry of the flow relative to the plane $\eta = 1$ does not require that the derivative $d\rho/d\eta$ vanish on it. An analogous requirement on the derivative $dv/d\eta$ follows not from the fact of symmetry in itself, but from the necessity that the stress vanish on this plane.

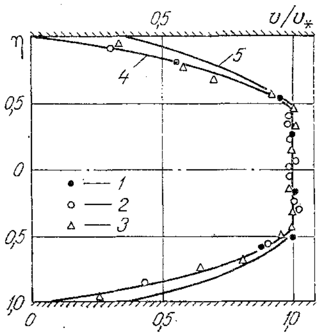

Fig. 4. Dimensionless velocity profiles in capillary with $\langle\rho\rangle = 0.34$; 1–3) experiment in [1] with $\varkappa = 0.056$ and total flow rates $Q = 0.711 \cdot 10^{-2}$; $3.56 \cdot 10^{-2}$ and $7.11 \cdot 10^{-2}$ cm^3/sec; 4, 5) theory for $m\varkappa = 0$; 0.2; $\rho_* = 0.50$; the value $\langle\rho\rangle = 0.34$ corresponds to $\rho_W \approx 0.175$ and $\eta_* \approx 0.45$

The dependences of ρ_W and η_* on $\langle\rho\rangle$, following from (24) with $\rho_* = 0.50$, are also presented in Fig. 1a.

Integrating the first equation in (21) using the conditions in (20), we have instead of (17) or (18) the relations

$$v(\eta) = \begin{cases} v_* = v_0 + \dfrac{1}{2}\displaystyle\int_{\eta_*}^{1} \frac{\eta d\eta}{M(\rho)}, & 0 \leqslant \eta < \eta_*, \\ v_0 + \dfrac{1}{2}\displaystyle\int_{\eta}^{1} \frac{\eta d\eta}{M(\rho)}, & \eta_* \leqslant \eta \leqslant 1, \quad v_0 = \dfrac{\varkappa}{2}\dfrac{k_w}{M_w}, \end{cases} \tag{25}$$

$$q = \pi\eta_*^2 v_* + 2\pi \int_{\eta_*}^{1} \eta v(\eta)\, d\eta.$$

The concentration and velocity profiles, described by Eqs. (25) and (23) at $\rho_* = 0.50$ and $\varkappa = 0$, are illustrated in Fig. 3.

The effective viscosity μ_e, measured in experiments with capillary viscosimeters, is expressed in terms of the dimensionless flow rate of the suspension q from (25) by the relation

$$\mu_e = \mu_0\pi/8q, \tag{26}$$

which replaces (19). The dependences μ_e/μ_0 as a function of $\langle\rho\rangle$, corresponding to k_W from (5), $\rho_* = 0.50$ and different values of $m\varkappa$, are presented in Fig. 1b.

It is easy to see that there are two basic competing factors, which cause μ_e to differ from the viscosity $\mu = \mu_0 M(\langle\rho\rangle)$ for a flow of a uniform suspension. First of all, the structure formation investigated leads to depletion of particles in regions of the flow near the wall, which decreases the observed effective viscosity. Second, it leads to the manifestation of pseudoplastic properties of the suspension, i.e., to the formation of zones in the core of the flow moving as a solid body, which, on the contrary, increases μ_e.

On the whole, for low average concentrations and for sufficiently fine particles, the concentration profile almost over the entire transverse cross section of the channel is nearly uniform, while the velocity profile is nearly parabolic; as $\langle\rho\rangle$ and $\varkappa$ increase, the velocity profile becomes increasingly blunt, going over into the velocity profile characteristic for a pseudoplastic medium. This behavior agrees with the experimental facts [1, 3].

Figure 4 illustrates the velocity profiles for flow in a capillary in the relative coordinates η, v/v_* for $\rho_* = 0.50$ and $\langle\rho\rangle = 0.34$ for different values of $m\varkappa$. The figure also presents some results of the experiment in [1]. Analysis of the data in Fig. 4, as well as other analogous data, shows the satisfactory correspondence between theory and experiment.*

From the analysis performed above it follows that the appearance of a nonuniform concentration distribution is due to the necessity of conserving angular momentum of the dispersed medium in the flow and, in addition, in the final analysis, a distribution is established for which the average flow of angular momentum vanishes. This opens up the fundamental possibility for actively affecting the structure of flows by artificially

* We emphasize that the indicated agreement with experiment concerns only velocity profiles in a one-dimensional flow of suspensions with neutral buoyancy of particles, but not concentration profiles, for which there are presently several contradictory facts. In particular, it is asserted in [1, 3] that the particle concentration distribution over the cross section of the channel is nearly uniform.

changing this flux. The latter can be achieved by placing the suspension of dipole particles in a specially formed external field, giving rise to independent rotation of particles due to the action of external force couples on them.

In conclusion, we note that above we neglected, first of all, inertial effects, giving rise to transverse migration of particles in nonuniform flows and, second, fluctuation effects, primarily translational and rotational Brownian motion. The first factor is important for suspensions of sufficiently large particles, especially if the particle density and the density of the dispersed medium differ, and the second is important for fine colloidal particles. It is clear that both factors can affect the form of the stationary concentration distributions and for this reason must be taken into account in future theory.

NOTATION

a, particle radius; c, velocity of the suspension; c_0, slipping velocity at the wall; e, tensor of the deformation rates; h, one-half the slit width or capillary radius; I, unit tensor; k, a coefficient in (4) and (5); M, function introduced in (1)-(3); m, a constant in (5); n, normal coordinate; p, pressure; q, dimensionless flow rate; V, one-half the relative velocity of plates in a Couette flow; v, dimensionless velocity; v_0, dimensionless slipping velocity; v_*, dimensionless velocity of a closely packed core; x and y, longitudinal and transverse coordinates; y_*, coordinate of the closely packed core; α, β, integration constants; Γ, a function introduced in (1)-(3); γ, tensor of angular stresses; $\eta = y/h$; $\eta_* = y_*/h$; $\varkappa = a/h$; μ_0, μ_e, viscosity of the liquid phase and the effective viscosity of the suspension; ρ, volume concentration of particles; $\langle\rho\rangle$, ρ_w, ρ_c, average concentration in the flow, the concentration at the wall, and the concentration at the center; ρ_*, concentration of the closely packed state; ρ_w^*, critical concentration at the wall, corresponding to the appearance of a closely packed core with flow in a planar gap; $\boldsymbol{\sigma}$, stress tensor; σ, a parameter in (14) and (15); and τ, tangential stress.

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SETTLING OF A BIDISPERSE SUSPENSION

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The influence of the fractional composition on the settling velocity of a bidisperse suspension is investigated theoretically. The average particle radius of a settling bidisperse suspension is calculated.

Monodisperse suspensions are rarely encountered in practical engineering, and a disperse phase usually consists of a mixture of particles of different sizes. Despite this, considerably fewer reports have been devoted to the investigation of the motion of polydisperse than of monodisperse suspensions.

The force of interaction between the liquid and disperse phases of a polydisperse suspension of moderate concentration was determined in [1] using rigorous statistical methods. Without additional considerations, however, one cannot determine from [1] the velocities of motion of the separate fractions needed to study the settling of a polydisperse suspension.

The settling of multifraction suspensions of fine particles of equal density was investigated theoretically in [2-6] on the basis of various assumptions about the form of the dependence of these velocities on the fractional composition and the total volumetric concentration of the disperse phase. Here, by analogy with the monodisperse case, the dependence of the settling velocities of the individual fractions on the fractional composition and the total concentration was assigned in [2, 3] in power-law form, where the porosity of the suspension served as the base while the exponent depended on the composition. A modified cell model was used for these purposes in [4], and data of [7] on the magnitude of the force of interaction between a filtering stream and a stationary, polydisperse granular bed were used in [5, 6]. The settling of bidisperse suspensions of particles of equal density at low Reynolds numbers was investigated experimentally in [2-5, 8-11].

Let us consider the uniform gravitational settling of a bidisperse suspension of moderate concentration. The continuous phase consists of an incompressible Newtonian liquid with a viscosity μ_0 and density d_0, while the disperse phase consists of a mixture of two fractions of spherical particles with radii a' and a'' and a density d_1. The volumetric concentrations of the particles a' and a'' and of the entire disperse phase are ρ', ρ'', and $\rho = \rho' + \rho''$, respectively. The Reynolds numbers characterizing both the flow over individual particles and the motion of the phases on the average are small compared with unity. For determinacy, let $a'' > a'$ and let the distribution of the concentrations ρ' and ρ'' be uniform.

Using [1], we represent the force of interaction of each fraction with the continuous phase, due to the action of viscous-friction forces, as the sum of two terms: the first, allowing for the constrained nature of the settling, coincides with that of [1]; the second characterizes the interaction between the fractions, due to the difference between the settling velocities of the fractions. Then the equations of conservation of mass and momentum describing the settling of the fractions have the form

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